and $2+1$; then $3^2.2731.(22366891).\sqrt{22366891}=4620$ limit of divisors of the form prime $8mnx+1$ and $8mnx+(6mn+1)=312x+1$ and $312x+235$, and they are 313, 547, 859, 937, 1171, 1249, 1483, 1873(2731)3121, 3433, 4057, 4603 to limit, none of which will divide the balance, hence 22366891 is prime.

$\therefore$ factors are $3^2\times2731\times22366891$.

No solution of Problem 33 has been received.

## PROBLEMS.

43. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find the series of integral numbers in which the sum of any two consecutive terms is the square of their difference.

44. Proposed by A. H. HOLMES, Box 963. Brunswick, Maine.

The hypotenuse of a right-angled triangle $ABC$, right-angled at $A$, is extended equally at both extremities so that $BE=CD$. Draw $AD$ and $AE$. Find integral values for all the lines in the figure thus made.

# AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

## NOTE ON AVERAGE AND PROBABILITY WITH REFERENCE TO THE SOLUTIONS OF PROBLEM 26, pp, 282-83, AND 327-28.

By ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington D. C.

I WILL remark at the outset that, unfortunately, mathematicians are not agreed as to the method of solving certain problems in Average and Probability. The difference of opinion in some cases relates to the interpretation of the meaning of the problem, and in others to the quantity that should be considered as the independent variable, and between what limits taken, and again as to whether the "points" are uniformly distributed along a certain line or over a certain surface, etc.

If points be uniformly distributed on a line, the *number* of points is proportional to the length of the line; and if points be uniformly distributed over a surface, the number of points is *proportional* to the area of the surface, etc.; but if the points be *not* uniformly distributed, then the line or surface can not be taken as a *true* measure of the number of points.

Problem 26. "Find the average of all right-angled triangles having a given hypotenuse."

Prof. Matz' first solution, p. 82, would be correct if he had taken the limits of $x$ from 0 to $h$ instead of from 0 to $\frac{1}{2}h\sqrt{2}$. He supposes one of the legs to increase uniformly from 0 to $\frac{1}{2}h\sqrt{2}$, or till the legs become equal; but this assumption does not give *all* the triangles because, while $x$ *increases* uniformly from 0 to $\frac{1}{2}h\sqrt{2}$, $\sqrt{(h^2-x^2)}$ does not decrease uniformly from $h$ to $\frac{1}{2}h\sqrt{2}$. The limits should be 0 and $h$, for if one leg varies uniformly from 0 to $h$ *all possible* right-angled triangles will be generated, and the number of the triangles will be proportional to $h$.

If he had taken $x$ from 0 to $h$ in his first solution, and $\theta$ from 0 to $\frac{1}{2}\pi$ instead of from 0 to $\frac{1}{4}\pi$ in the second, he would have obtained in both solutions the result $\frac{1}{6}a^2$, which I believe to be correct.

In the second method of solution, adopted by Prof. Zerr and others, and approved by the Editor, it is assumed that one of the *acute angles* varies uniformly, and that the *number* of triangles is proportional to the semicircumference whose diameter is the given hypotenuse.

The vertices of the right angles of all possible right-angled triangles having a given hypotenuse $a$ will be posited on a semicircumference whose diameter is $a$, but *will not* be uniformly distributed thereon; hence the semicircumference *can not* be taken as the true measure of the number of triangles.

I most emphatically dissent from the conclusion announced by the Editor in the last line of p. 328. The last paragraph of the note is sound down to the last line, but it does not by any means necessarily follow from any statement made therein that "the solutions leading to the result $\frac{a^2}{2\pi}$ are the *correct* and *only* solutions of the problem." I hold that the solution given by Prof. Anthony on p. 283, and previously given by myself in the *Mathematical Magazine*, leading to the result $\frac{1}{6}a^2$ (misprinted $\frac{1}{3}a^2$ in the first line of the Editor's note), is the *true* solution of the problem.

The conception of a triangle is from its sides; and if we cause one of the legs to take all possible values from 0 to $a$ it is very clear to me (and ought to be to every one) that all possible right-angled triangles having that hypotenuse will be formed.

The problem as proposed is *definite* as the Editor correctly states in his note, and requires the "average area of *all* the right-angled triangles having a given hypotenuse"; but the solution which he asserts the "*correct* and *only*" one restricts the triangles to those having the vertices of their right angles uniformly distributed on the semicircumference whose diameter is the given hypotenuse, and therefore is not a solution of the problem proposed, but of the following problem, viz: Required the average area of the right-angled triangles having a given hypotenuse and the vertices of their right angles uniformly distributed on the semicircumference whose diameter is the given hypotenuse.